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Role of Time in the Sum-over-Histories Framework for Gravity

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I sketch a self-contained framework for quantum mechanics based on its path-integral or "sum-over-histories" formulation. The framework is very close to that for classical stochastic processes like Brownian motion, and its interpretation requires neither "measurement" nor "state-vector" as a basic notion. The rules for forming probabilities are nonclassical in two ways: they use complex amplitudes, and they (apparently unavoidably) require one to truncate the histories at a "collapse time," which can be chosen arbitrarily far into the future. Adapting this framework to gravity yields a formulation of quantum gravity with a fully "spacetime" character, thereby overcoming the "frozen nature" of the canonical formalism. Within the proposed adaptation, the value of the "collapse time" is identified with total "elapsed" spacetime four-volume. Interestingly, this turns the cosmological constant into an essentially classical constant of integration, removing the need for microscopic "fine tuning" to obtain an experimentally viable value for it. Some implications of the "$V = T$" rule for quantum cosmology are also discussed.

It has often been remarked that without the one-dimensional "invariance group" of time translations, science as we know it would be impossible because the outcome of an experiment would depend on when it happened to be performed. In this sense the gauge concept can be traced back to the earliest times, and the history of gauge theories is the history of science itself. After being born in this way—and of course after a lapse of many years—the simple invariance group of time translations relin-

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quished its independent identity by entering into the Galilean group, where it could no longer be uniquely separated from the spatial translations. Still later the synthesis with spatial symmetry became more complete when the Poincaré group replaced the Galilean one. And finally, in the diffeomorphism group, the symmetry group of general relativity, all traces of a separate meaning for time have disappeared, with any smooth reshuffling of spacetime points being admissible.

Now, *classically* this vast enlargement of the gauge group has been a wholly satisfactory development. In denying genuine individuality to the points of spacetime, general covariance is telling us that these points exist not for themselves, but only as *carriers* for the metric, and for the other, "matter" fields that interact with the metric.

Incidentally, why are not *all* permutations of spacetime points included in the general covariance group; why do we limit ourselves to rearrangements which are continuous and smooth? The answer of course is that the underlying spacetime manifold M is not entirely formless. Its points carry not only the metric and "matter" fields, but first of all a topology and a differentiable structure. But these elements of structure, unlike the fields they support, remain (classically) "absolute" ($\equiv$background $\equiv$ nondynamical). Hence, every symmetry must preserve them.

For us, though, the important thing is that *time* has no place among these elements of absolute relativistic structure, pertaining rather to the particular metric g_{ab} with which M happens to be endowed. Conversely, M deprived of any specific metric possesses no notion of time and therefore no remnant of the distinction between spacelike and nonspacelike hypersurfaces. Thus, time is *not* one of the background structures of classical relativistic physics.

In contrast, *quantum* dynamics has so far proved impossible to formulate except with the aid of a background structure representing time. Indeed, I know of only three methods of formulating quantum theories, and in each of them this distinguished role of time can be seen clearly. In *canonical* formulations, the basic dynamical equation is the Schrödinger equation, $H\varphi = i\hbar\, \partial\psi/\partial t$, in which t appears glaringly. In what I believe are called "covariant" formulations (i.e., dynamics expressed as covariant field equations relating operators at different spacetime points) one invokes equal *time* commutation relations to supplement the equations of motion in generating the complete set of algebraic relations among the field operators. Finally, there is the sum-over-histories formulation, to which I will return shortly. As it is most commonly interpreted, namely as a technique to produce transition amplitudes $\langle q_2 t_2 | q_1 t_1 \rangle$, it is not really an alternative framework, and anyway one still requires some background time with respect to which the labels t_1 and t_2 can make sense (unless the dependence on t were to drop out, which is precisely what happens in most forms of "canonical quantum gravity" for a reason I will discuss in a moment).[2]

So we encounter a contradiction between general convariance on one hand and the quantum need for some form of background time on the other hand.

Let us see briefly how this contradiction manifests itself in the canonical formulations of quantum gravity. There we have a spacelike hypersurface $\mathscr{H}(t)$ advancing through M in the parameter-time t. The spacetime metric g is replaced by a path $q(\cdot)$, where $q(t) = g|\mathscr{H}(t)$ is the restriction of g to $\mathscr{H}(t)$. However, nothing physical is affected by a reparametrization $t \to t' \equiv f(t)$. Since in particular the action is not changed, the generator of δt, that is to say the Hamiltonian, must vanish, leading to the Schrödinger equation

$$i\frac{\partial\psi}{\partial t} = H\psi = 0 \cdot \psi = 0$$

Thus the wavefunction ψ, which *a priori* depended on q and t, turns out to be a function of q only: spacetime collapses to just space!

In the resulting formalism the general covariance group is not realized, but only the group of diffeomorphisms of a single hypersurface. All the other diffeomorphisms get entangled with the dynamics, resulting additionally in the problems that the Hilbert space inner product is difficult or impossible to define, and the physical observables difficult or impossible to recognize.

But, even apart from all these "technical" problems, the inability to refer directly to spacetime makes it hard to formulate questions with physical meaning. For example, a black hole horizon is by its very nature

[2]At about this point in my talk some questions and objections were raised concerning the assertion that time plays a distinguished role in quantum dynamics. One person pointed out that the hypersurfaces to which the parameter t refers can be any spacelike ones, and therefore covariance does not seem to be lost. However, this remark referred to flat space, where the background Minkowski metric has *already* broken the diffeomorphism group down to the Poincaré group. There, the remaining Lorentz covariance is indeed not lost, because none of its transformations turn a spacelike hypersurface into a nonspacelike one. More generally, not only flat-space theories, but even quantum field theory in curved spacetime is still OK, because the metric—if globally hyperbolic—still allows the appropriate initial-value surfaces to be recognized. However, with the advent of *general* covariance it becomes impossible to any longer distinguish *a priori* spacelike hypersurfaces from ones which are timelike somewhere. Another participant remarked that "stochastic quantization" offers a fourth formulation of quantum dynamics. For my purposes, however, I think it can be viewed as a species of sum-over-histories formalism, or more exactly as a tool to compute the (Euclidean) path integral, whose physical interpretation remains unaffected.

defined only with respect to the global geometry, and a universe which expands, contracts, and then "bounces" into a reexpansion would seem impossible to describe in terms of only some time-independent wavefunction $\psi(q)$. Thus, questions about the behavior of an evaporating black hole or a collapsing universe become nearly impossible to ask, as do also, for example, questions about the probability of a strong gravitational field creating a pair of (topological) geons.

More generally, the notion of a 4-dimensional space*time* seems recoverable within a canonical formalism only in the semiclassical limit, which is the regime of least interest from the point of view of quantum gravity. (Incidentally, the conflict with diffeomorphism invariance is even worse for string theory, which, at least up until now, destroys even more of general covariance by introducing a background spacetime metric.)

A formulation of quantum dynamics that seems at first sight to escape from the contradiction between time and general covariance is what I will call the sum-over-histories framework. To understand where this formulation comes from, let us consider first the description of Brownian motion as a (purely classical) stochastic process. Here the physical reality is a *path* (or "history") $\gamma = \gamma(t)$, and the meaningful questions refer in general to properties of the path as a whole. However, the theory, though classical, is inherently nondeterministic and furnishes nontrivial answers only to questions of the form: "With what probability P does the actual path lie in such and such a (measurable) subset C of the space of all possible paths?"

A typical question of this kind is, "Will the particle return to the origin?," to which the answer is, "Yes with probability $P(C|I)$," where $C = \{\gamma | \gamma(t) = 0 \text{ some } t > 0\}$, and I have assumed the initial condition I that $\gamma(0) = 0$. The dynamical information is thus contained in the rule for computing P, which formally is just

$$P(C|I) = \sum_{\gamma \in C} p(\gamma|I) \tag{1}$$

where the $p(\gamma)$ are positive real numbers weighting the individual paths. Notice here that the question C refers to paths which are *infinite* into the future, and that $P(C)$ is only very indirectly related to "transition probabilities" of the form $p(x_2 t_2 | x_1 t_1)$. Note especially that what is physically real is the actual path γ, and not some probability density $\rho(q, t)$ to find the particle at position q at time t.

Now the quantum mechanical sum-over-histories framework is almost wholly taken over from that just sketched. The "only" difference is that the probabilities $P(C|I)$ are computed according to a different rule from (1), a rule involving complex amplitudes rather than just positive probability weights.

To interpret the sum over histories as I have just done potentially helps quantum gravity in two ways. First, it *appears* to do without any notion of distinguished time; and second, it makes no direct reference to "measurements" as basic notions, thereby protecting us from uncertainty principle arguments that the metric cannot possibly be measurable at the relevant (Planck) scales. It also brings the philosophical improvement that there is no "state vector collapse," because there is no state vector *to* collapse, γ itself being real rather than some wavefunction $\psi(q, t)$. However, this last aspect has more to do with quantum theory in general than with its relation to gravity.

Now what rule for obtaining $P(C|I)$ does quantum mechanics put in place of equation (1)? As a first approximation let us suppose the rule is as follows: Assign to each path γ some complex amplitude $A(\gamma)$ and set

$$P(C) = |A(C)|^2 \tag{2a}$$

where

$$A(C) = \sum_{\gamma \in C} A(\gamma) \tag{2b}$$

is the sum of the amplitudes of all the paths comprising C. Here $P(C)$ is the relative probability of C and must be normalized by reference to the alternative subsets $D, E, F, \ldots$ from which C is being distinguished. If this rule were true, then—except for the final squaring of A to get P—quantum probabilities would be just like classical ones, only with complex weights replacing positive real ones. In particular, probabilities could be computed without ever introducing a distinguished time, because amplitudes would be functions of whole paths, or whole spacetimes in the case of gravity.

Unfortunately this appealingly simple picture cannot—as far as I can see—be maintained, and therefore it must presumably be modified also for gravity. Specifically, it has two failings, the first of which is that the correct amplitudes $A(C)$ seem *not* to be always additive as (2b) would imply. The second problem is that the amplitudes $A(\gamma)$ cannot really be chosen freely if any semblance of locality and causality is to be maintained. Unfortunately I have no time to explain in detail what I mean by this second remark except to say that causality requires that the performance of future measurements or the introduction of external fields in the future should not affect the probabilities for earlier events.

Let us, however, consider further the question of additivity in the case, say, of a single point particle moving in a potential $V = V(x)$. Assume the initial condition $q = q_0$ at $t = t_0$, and suppose that the finite set of positions $q_1 = q(t_1), q_2 = q(t_2) \ldots q_n = q(t_n)$ are to be measured at times $t_1, t_2, \ldots, t_n$.

The set of all paths γ is thereby sorted into subsets characterized by the values $q_1, q_2, \ldots$ and one can check that the probability of the particular subset $C(q_n, \ldots, q_2, q_1|q_0)$ is given by

$$P(C) = |\langle q_n t_n | q_{n-1} t_{n-1} \rangle \cdots \langle q_2 t_2 | q_1 t_1 \rangle \langle q_1 t_1 | q_0 t_0 \rangle|^2$$

in terms of the usual matrix elements $\langle q'' t'' | q' t' \rangle$. It is thus natural to assume that $A(C)$ is the expression inside the vertical bars, i.e., to assume, for example, that

$$A(2, 1|0) = \langle 2|1 \rangle \langle 1|0 \rangle \tag{3}$$

where I have used obvious abbreviations for $A(C(q_n, \ldots, q_2, q_1|q_0))$, $\langle q_2 t_2 | q_1 t_1 \rangle$, etc.

If we accept this form for $A(C)$, additivity can first of all be applied to the relation

$$C(2|0) = \bigcup_1 C(2, 1|0)$$

to yield

$$A(2|0) = \sum_1 A(2, 1|0)$$

or

$$\langle 2|0 \rangle = \sum_1 \langle 2|1 \rangle \langle 1|0 \rangle$$

which is indeed true by "completeness." However, it is equally true at the level of subsets that

$$C(1|0) = \bigcup_2 C(2, 1|0)$$

but this time the corresponding relation among amplitudes fails:

$$\sum_2 \langle 2|1 \rangle \langle 1|0 \rangle \neq \langle 1|0 \rangle$$

because $\sum_2 \langle 2|1 \rangle \neq 1$ in general (rather $\sum_2 |\langle 2|1 \rangle|^2 = 1$, as we know).

Although the rule (2) seems to have failed, there does exist a rule which will reproduce the correct relative probabilities for the classes $C(n, \ldots, 2, 1|0)$ described above. To state this rule we must first choose an arbitrary time T to the future of all the times $t_1, t_2, \ldots, t_n$ (I will call T the "collapse time"), and compute amplitudes only for paths truncated at $t = T$, i.e., for paths $\gamma(t)$ defined for $t \in [t_0, T]$. Second, we must choose an arbitrary q and define instead of (2b),

$$A(q, T; C|I) = \sum_{\substack{\gamma \in C \\ \gamma(T) = q}} A(\gamma) \tag{4}$$

to be the total amplitude of all truncated paths in C that arrive at q at the collapse time T. Finally we put in place of (2a)

$$P(C|I) = \sum_q |A(q, T; C|I)|^2 = \sum_q \left| \sum_{\substack{\gamma \in C \\ \gamma(T) = q}} A(\gamma) \right|^2 \tag{5}$$

The resulting probabilities then agree with the standard quantum mechanical ones for any sequence of observations at times $t_1, \ldots, t_n$ each of whose operators is a function of the position operator at that time. In particular the probabilities come out correct for the sequence of complete position measurements considered above.

In comparison with (2), the rule (5) involves a somewhat unnatural looking combination of coherent and incoherent summation; but the main disappointment is that an explicit reference to time has returned in the form of the auxiliary parameter T which appears in (4) and (5). Moreover, the freedom to choose $A(\gamma)$ is further limited by the consistency condition that $P(C|I)$, which now appears to depend on T, be in fact independent of it. A sufficient condition for this, if A has the form considered in (3), is that

$$\sum_3 \langle 3|2\rangle^* \langle 3|1\rangle = \delta(2, 1)$$

where $t_3 > t_2 > t_1$. It is thus natural to view the condition that T drop out of (5) as a generalized form of unitarity. Our general dynamical rule is then one which, in a certain sense, has "retreated back toward a Schrödinger formulation," insofar as both time in some form and unitary evolution in some form figure essentially in its formulation. Nevertheless, the retreat is only partial, and enough of the spacetime character of (1) remains that we still may hope to find in the sum over histories a vehicle for formulating a physically consistent dynamical framework for quantum gravity.

As adapted to gravity, the sum-over-histories method should provide, for appropriate classes of 4-geometries, relative probabilities computed according to some rule analogous to (4). To frame such a rule, we need to find analogs of γ, T, and q, and to choose a specific expression for $A(\gamma)$. For the truncated path γ, the obvious analog is a compact spacetime manifold M, with metric $g = g_{ab}$ and future boundary ∂M. (A past boundary, if any, will occur only in conjunction with initial conditions. Ultimately, any fundamental boundary conditions must be cosmological in character; for example, the universe might be supposed to have originated from a condition of zero volume. For present purposes, let me just ignore the question of initial conditions, ignoring correspondingly any past boundary that M might possess.) For q, the analog nearest at hand is the induced metric on ∂M: $q = g|\partial M$; and for $A(\gamma)$ one will of course take $A = Re^{iS}$, where S is the gravitational action (including surface terms) and R represents the functional-integral "measure."

There remains the "collapse time" T, whose gravitational analog is far from obvious. Indeed, it may even be questioned whether any analog of the restriction [in (4)] to paths of temporal length T *should* be imposed; but if it is not, then I believe one may expect both technical and physical problems with the resulting theory. Specifically, I am thinking of problems with the convergence of the sum analogous to (5), with the recovery in the "semiclassical" limit of ordinary quantum field theory on curved spacetime (which *does* involve T), and with the "causality condition" that (the performance of) future measurements should not affect present probabilities. Let us accept, then, that an analog of the restriction to histories of temporal length T is in fact needed in quantum gravity.

What is this analog? In the remainder of this paper, I will briefly explore some consequences of identifying it with the condition $V(g) = T$, where $V(g)$ is the total space*time* volume of the "history" γ, i.e., the four- (or whatever-) dimensional volume

$$V(g) = \int_M \sqrt{-g}\, dx$$

Apart from the fairly natural character of this proposal, it has also the major virtue (in my mind) of remaining meaningful when continuous geometries are replaced by "causal sets," which I personally would like to believe are the discrete substratum of spacetime.[3] For a causal set, V becomes simply N, the total number of elements making up the set. Thus our condition can be imposed on arbitrary causal sets, including ones far from being approximatable by any continuum geometry (M, g). For present purposes, however, I will remain within the continuum context, and not consider possible further modifications to our rules that might be mandated by considering (M, q) to be merely an approximation to some causal set or class of such sets.

With the adoption of $V(q)$ as the measure of "time," the formula analogous to (5) for the relative probability of a class C of histories becomes

$$P(C|I) = \sum_q \left|\sum_\gamma{}' A(\gamma)\right|^2 \tag{6}$$

where the "primed" inner sum is over all geometries $\gamma = (M, q) \in C$ which obey in addition the two conditions $g|\partial M = q$ and $V(g) := \int_M \sqrt{-g}\, dx = T$.

[3]The notion of causal set is explained in Bombelli *et al.* (1987).

As in the nonrelativistic case, the question immediately arises whether the probability defined in (6) is independent of T. Unfortunately, I do not know the general answer to this question, which of course can only really acquire meaning with respect to some regularization-scheme-cum-choice-of-"measure" for defining the right-hand side of (6). At any rate the answer is "yes" in a very special case that we will examine shortly: it is not hard to ensure "unitarity" for the simplest minisuperspace examples, such as the three types of locally homogeneous and isotropic cosmology.

Let me break off this general discussion of the "$V = T$" rule and turn to a brief consideration of some of its most striking consequences: the reinterpretation of the cosmological constant as a *freely adjustable classical parameter* analogous to a constant of integration; and a modified quantum cosmology with genuine time dependence and an unexpectedly strong influence of the spatial curvature on behavior near the zero-volume singularity.

In speaking just now of the cosmological constant, I of course meant the *observed* parameter Λ, which is macroscopic, classical, and smaller in magnitude than about 10^{-120} in natural units. To understand the meaning of such a classical parameter, we should first recall how the classical limit in general is realized within the sum-over-histories framework, namely as a stationary phase approximation ($\hbar \to 0$ in $A = Re^{iS/\hbar}$) that selects as the "classical histories" those for which $\delta S = 0$. Now the only phases which matter are those that lead to interference in the inner ("coherent") sum in (6). But since this sum extends only over metrics of fixed volume $V = T$, the variation δS need vanish *only* for variations δg that preserve the total spacetime volume V. In other words, the criterion for a classical path is

$$\delta(S - \lambda V) = 0$$

where λ is a Lagrange multiplier. Inasmuch as any *microscopic* cosmological constant Λ_0 that might be present just contributes a term $-\Lambda_0 V$ to the action, we see that the classical limit is governed by the effective cosmological constant

$$\Lambda = \Lambda_0 + \lambda$$

But since λ is freely choosable, so also is Λ, and nothing in the microscopic theory can determine its value!

This is an improvement over theories that almost inevitably produce the *wrong* value of Λ, but of course it does not yet tell us why nature uses this new found freedom to adjust Λ to a value so near zero. In fact, I suspect that no continuum theory can answer this question, although a theory based on causal sets might. Indeed there are indications that such a theory might predict $\Lambda \sim V^{-1/2}$, which would put Λ just at the limit of present observation, but the argument is so undeveloped (and the time remaining for discussion so short), that I had better not go into it here.

Finally, let us consider the implications of the $V = T$ rule for the simplest conceivable "quantum cosmologies," namely the matter-free "Friedmann universes" with scale factor a and compact spatial geometry locally isometric to one of the three homogeneous-isotropic spaces S^{n-1}, $\mathbb{R}^{n-1}$, H^{n-1} (i.e., $M = \mathbb{R} \times K$, where K is a quotient of one of these three spaces). In this case the question of "unitarity" reduces to the question of whether the analog of (4), which I will call $\psi(a, T)$, has an L^2-norm which is independent of T (as long as T is big enough that the conditions defining C are to its past). Now the "path integral" in (6) implies in the usual way a Schrödinger-like equation for ψ:

$$i \frac{\partial \psi}{\partial T} = H\psi$$

where H is formally the Hamiltonian operator derived from the Lagrangian,

$$L = -c_1 \left(\frac{dv}{dT}\right)^2 + \frac{c_2}{v^{2/d}} - \Lambda_0$$

Here I have made a change of variables from (a, T) to (v, T), where v is the total *spatial* volume "at 'time' T"; $n \equiv d + 1$ is the spacetime dimension (assumed ≥ 2); Λ_0 is what I called before the "microscopic cosmological constant"; and the constants c_1 and c_2 are defined as follows in terms of the "gravitational constant" κ [normalized so that the action $S = (1/2\kappa) \int R\, dV + \cdots$] and the numerical constant k, depending only on the spatial topology, defined by the equality (scalar curvature of spatial metric) $= kv^{-2/d}$:

$$c_1 = \kappa^{-1}(n/2 - 1)/(n - 1), \qquad c_2 = k/2\kappa$$

Notice that with v and T as basic variables, the "kinetic energy" term in L becomes that of a particle of constant (though negative) mass.

From the expression (7) it follows first of all that, with suitable choices of measure $\rho(v)\, dv$, of "operator ordering," and of boundary conditions for ψ, H will be self-adjoint, in which case the norm of ψ will indeed be independent of T. Thus our unitarity condition can be fulfilled in this case, as I claimed earlier [although one might not actually want to fulfill it if the universe were allowed to "collapse and disappear," but then our rule (6) would no longer be adequate in its present form either].

The second interesting observation we can make apropos of (7) is that the sign of the "potential term" in L depends on the sign of the spatial curvature. For the spherical case (k positive) there is a *repulsive* singularity at $v = 0$ (it looks attractive; but remember that the kinetic energy is also reversed in sign—a consequence of its representing a conformal degree of freedom), while for hyperbolic case (k negative) there is an attractive singularity of the same strength. This suggests that the behavior of a recollapsing (or even an initially expanding) universe could be very different in the three cases $k > 0$, $k < 0$, $k = 0$. (Note, however, that a change of operator ordering in the kinetic term of H could modify this conclusion by introducing terms in v^{-2} into the potential.)

A third observation is that Λ_0 functions here as an irrelevant additive constant in the "Hamiltonian" H. Obviously this is closely connected to our earlier observation that in general the cosmologically observable Λ is independent of the parameter Λ_0 that appears in the action.

It is also connected to the biggest conceptual change that has occurred here relative to versions of quantum cosmology which impose the Wheeler–DeWitt equation $H\Psi = 0$ on Ψ. As we know, time is "frozen" in such treatments; but here there is genuine change with T, and Ψ might, for example, be a wave packet rather than a "stationary state" whose temporal implications must be inferred indirectly via a WKB approximation, or by some other method. Although in the present interpretation Ψ is only an auxiliary quantity, an aid in evaluating some $P(C|I)$, its T dependence is nevertheless significant. It means, I think, that the active, dynamical aspect of quantum mechanics has managed to assert itself, even in a situation where time has been so thoroughly geometrized and "spatialized" as it is in general relativity.

In concluding, I would not want to have given the impression that I think the sum-over-histories framework is complete in itself. Rather, I think the physical meaning of probabilities like $P(C|I)$ is far from being settled. Specifically, what is the meaning of dividing the set of all histories into classes $C_1, C_2, \ldots, C_n$ and then computing a probability for each class? Does such a separation make sense in itself, or does it refer implicitly to some measuring apparatus which enforces physically the separation of each class C_i from the others? If the former, then how objective are the probabilities we compute; if the latter, then how can we do quantum cosmology? Such questions call for a theory of measurement, which so far has hardly been developed within the sum-over-histories framework.

A circle of issues intimately related to those just raised involves the contrast between the intrinsically global definition of amplitudes and the physical locality and causality that are so characteristic of known systems. In operator formulations with "state vector collapse" causality and locality can be built in naturally (via unitary evolution and spacelike commutability), but in the sum-over-histories framework they arise in a less transparent manner, a manner that I feel is not adequately understood, although it is clearly tied to the specific form taken by (5) and (6). Especially in gravity, where the status of causality and locality is least clear, one can hope that further study of the origin and meaning of these twin properties will help us understand whether formulations like the one given above are adequate, or whether a different (possibly more radical) way of incorporating time is needed.

Added Note. My views on the sum-over-histories framework have evolved since this talk was delivered in 1987; see Sorkin (1991), Sorkin and Sinha (1991), and Sorkin (1993). Were I writing today, I would present the question of whether to impose $V = T$ as basically a technical issue, without viewing T therein as more than a formal analog to the time-parameter which occurs in the Schrödinger equation.

Since this paper was written, several works proposing "$V = T$" in a canonical framework have appeared (e.g., Unruh and Wald, 1989; Henneaux and Teitelboim, 1989; Brown and York, 1989).

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